

## ИДЗ «Дифференциальные уравнения высших порядков»

**Задача 1.** Найти частное решение дифференциального уравнения.

1.1  $y''' = \sin x, \quad y(0) = 1, y'(0) = 0, y''(0) = 0.$

1.2  $y''' = 1/x, \quad y(1) = 1/4, \quad y'(1) = y''(1) = 0.$

1.3  $y'' = 1/\cos^2 x, \quad y(0) = 1, \quad y'(0) = 3/5.$

1.4  $y''' = 6/x^3, \quad y(1) = 0, \quad y'(1) = 5, \quad y''(1) = 1.$

1.5  $y'' = 4 \cos 2x, \quad y(0) = 1, \quad y'(0) = 3.$

1.6  $y'' = 1/(1+x^2), \quad y(0) = 0, \quad y'(0) = 0.$

1.7  $xy''' = 2, \quad y(1) = 1/2, \quad y'(1) = y''(1) = 0.$

1.8  $y''' = e^{2x}, \quad y(0) = \frac{9}{8}, \quad y'(0) = \frac{1}{4}, \quad y''(0) = -\frac{1}{2}.$

1.9  $y''' = \cos^2 x, \quad y(0) = 1, \quad y'(0) = -1/8, \quad y''(0) = 0.$

1.10  $y'' = 1/\sqrt{1-x^2}, \quad y(0) = 2, \quad y'(0) = 3.$

1.11  $y'' = \frac{1}{\sin^2 2x}, \quad y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}, \quad y'\left(\frac{\pi}{4}\right) = 1.$

1.12  $y'' = x + \sin x, \quad y(0) = -3, \quad y'(0) = 0.$

1.13  $y'' = \operatorname{arctg} x, \quad y(0) = y'(0) = 0.$

1.14  $y'' = \operatorname{tg} x \cdot \frac{1}{\cos^2 x}, \quad y(0) = 1/2, \quad y'(0) = 0.$

1.15  $y''' = e^{x/2} + 1, \quad y(0) = 8, \quad y'(0) = 5, \quad y''(0) = 2.$

1.16  $y'' = x/e^{2x}, \quad y_0(0) = 1/4, \quad y'(0) = -1/4.$

1.17  $y'' = \sin^2 3x, \quad y(0) = -\pi^2/16, \quad y'(0) = 0.$

1.18  $y''' = x \sin x, \quad y(0) = 0, \quad y'(0) = 0,$

1.19  $y'' = \cos x + e^{-x}, \quad y(0) = -e^{-\pi}, \quad y'(0) = 1.$

1.20  $y''' = \sqrt{x} - \sin 2x, \quad y(0) = -1/8, \quad y'(0) = \frac{1}{8} \cos 2, \quad y''(0) = \frac{1}{2}.$

1.21  $y'' = \frac{1}{\cos^2(x/2)}, \quad y(0) = 0, \quad y'(0) = 1.$

1.22  $y'' = 2 \sin x \cos^2 x, \quad y(0) = -5/9, \quad y'(0) = -2/3.$

1.23  $y'' = 2 \sin^2 x \cos x, \quad y(0) = 1/9, \quad y'(0) = 1.$

1.24  $y'' = 1/x^2, \quad y(1) = 3, \quad y'(1) = 1.$

$$1.25 \quad y'' = x - \ln x, \quad x_0 = 2, \quad y(1) = -5/12, \quad y'(1) = 3/2.$$

$$1.26 \quad y''' \sin^4 x = \sin 2x, \quad x_0 = 5\pi/2, \quad y(\pi/2) = \pi/2, \\ y'(\pi/2) = 1, \quad y''(\pi/2) = -1.$$

$$1.27 \quad y'' = \frac{1}{\sqrt{4-x^2}}, \quad y(0) = 2, \quad y'(0) = 3.$$

**Задача 2.** Найти общее решение дифференциального уравнения.

$$2.1 \quad y'''x \ln x = y''.$$

$$2.2. \quad xy''' + y'' = 1.$$

$$2.3 \quad 2xy''' = y''.$$

$$2.4. \quad xy''' + y'' = x + 1.$$

$$2.5 \quad \operatorname{tg} x \cdot y'' - y' + \frac{1}{\sin x} = 0.$$

$$2.6. \quad x^2 y'' + xy' = 1.$$

$$2.7. \quad y''' \operatorname{ctg} 2x + 2y'' = 0.$$

$$2.8. \quad x^3 y''' + x^2 y'' = 1.$$

$$2.9. \quad \operatorname{tg} x \cdot y''' = 2y''.$$

$$2.10. \quad y'' + \frac{2x}{x^2+1} y' = 2x.$$

$$2.11. \quad x^4 y'' + x^3 y' = 1.$$

$$2.12. \quad xy''' + 2y'' = 0.$$

$$2.13. \quad (1+x^2)y'' + 2xy' = x^3.$$

$$2.14. \quad x^5 y''' + x^4 y'' = 1.$$

$$2.15. \quad xy''' - y'' + \frac{1}{x} = 0.$$

$$2.16. \quad xy''' + y'' + x = 0.$$

$$2.17. \quad x^4 y'' + x^3 y' = 4.$$

$$2.18. \quad xy''' + y'' = \sqrt{x}.$$

$$2.19. \quad y''' \operatorname{tg} x = y'' + 1.$$

$$2.20. \quad y''' \operatorname{tg} 5x = 5y''.$$

$$2.21. \quad xy''' + y'' = \frac{1}{\sqrt{x}}.$$

$$2.22. \quad x^3 y''' + x^2 y'' = \sqrt{x}.$$

$$2.23 \quad (x+1)y''' + y'' = (x+1).$$

$$2.24. \quad (1 + \sin x)y''' = \cos x \cdot y''.$$

$$2.25 \quad (1 + \sin x)y''' = \cos x \cdot y''.$$

$$2.26 \quad xy''' + y'' = \frac{1}{\sqrt{x}}.$$

$$2.27 \quad -xy''' + 2y'' = \frac{2}{x^2}.$$

**Задача 3.** Найти общее решение дифференциального уравнения.

$$3.1 \quad \text{а) } y'' + 4y = 0; \quad \text{б) } y'' - 10y' + 25y = 0; \quad \text{в) } y'' + 3y' + 2y = 0.$$

$$3.2 \quad \text{а) } y'' - y' - 2y = 0; \quad \text{б) } y'' + 9y = 0; \quad \text{в) } y'' + 4y' + 4y = 0.$$

- 3.3 а)  $y'' - 4y' = 0$ ; б)  $y'' - 4y' + 13y = 0$ ; в)  $y'' - 3y' + 2y = 0$ .
- 3.4 а)  $y'' - 5y' + 6y = 0$ ; б)  $y'' + 3y' = 0$ ; в)  $y'' + 3y' + 2y = 0$ .
- 3.5 а)  $y'' - 2y' + 10y = 0$ ; б)  $y'' + y' - 2y = 0$ ; в)  $y'' - 2y' = 0$ .
- 3.6 а)  $y'' - 4y = 0$ ; б)  $y'' + 2y' + 17y = 0$ ; в)  $y'' - 4y' - 12y = 0$
- 3.7 а)  $y'' + y' - 6y = 0$ ; б)  $y'' + 9y' = 0$ ; в)  $y'' - 4y' + 20y = 0$
- 3.8 а)  $y'' - 49y = 0$ ; б)  $y'' - 4y' + 5y = 0$ ; в)  $\phi'' + 8\phi' + 16\phi = 0$
- 3.9 а)  $y'' + 7y' = 0$ ; б)  $y'' - 5y' + 4y = 0$ ; в)  $y'' + 16y = 0$
- 3.10 а)  $y'' - 6y' + 8y = 0$ ; б)  $y'' + 4y' + 5y = 0$ ; в)  $y'' + 5y' = 0$ .
- 3.11 а)  $4y'' - 8y' + 3y = 0$ ; б)  $y'' - 3y' = 0$ ; в)  $y'' - 2y' + 10y = 0$ .
- 3.12 а)  $y'' + 4y' + 20y = 0$ ; б)  $y'' - 3y' - 10y = 0$ ; в)  $y'' - 16y = 0$ .
- 3.13 а)  $9y'' + 6y' + y = 0$ ; б)  $y'' - 4y' - 21y = 0$ ; в)  $y'' + y = 0$ ;
- 3.14 а)  $2y'' + 3y' + y = 0$ ; б)  $y'' + 4y' + 8y = 0$ ; в)  $y'' - 6y' + 9y = 0$ .
- 3.15 а)  $y'' - 10y' + 21y = 0$ ; б)  $y'' - 2y' + 2y = 0$ ; в)  $y'' + 4y' = 0$ .
- 3.16 а)  $y'' + 6y' = 0$ ; б)  $y'' + 10y' + 29y = 0$ ; в)  $y'' - 8y' + 7y = 0$ .
- 3.17 а)  $y'' + 25y = 0$ ; б)  $y'' + 6y' + 9y = 0$ ; в)  $y'' + 2y' + 2y = 0$ .
- 3.18 а)  $y'' - 3y' = 0$ ; б)  $y'' - 7y' - 8y = 0$ ; в)  $y'' + 4y' + 13y = 0$ .
- 3.19 а)  $y'' - 3y' - 4y = 0$ ; б)  $y'' + 6y' + 13y = 0$ ; в)  $y'' + 2y' = 0$ .
- 3.20 а)  $y'' + 25y' = 0$ ; б)  $y'' - 10y' + 16y = 0$ ; в)  $y'' - 8y' + 16y = 0$ .
- 3.21 а)  $y'' - 3y' - 18y = 0$ ; б)  $y'' - 6y' = 0$ ; в)  $y'' + 2y' + 5y = 0$ .
- 3.22 а)  $y'' - 6y' + 13y = 0$ ; б)  $y'' - 2y' - 15y = 0$ ; в)  $y'' - 8y' = 0$ .
- 3.23 а)  $y'' + 2y' + y = 0$ ; б)  $y'' + 6y' + 25y = 0$ ; в)  $y'' - 4y' = 0$ .
- 3.24 а)  $y'' + 10y' = 0$ ; б)  $y'' - 6y' + 8y = 0$ ; в)  $4\phi'' + 24\phi' + \phi = 0$
- 3.25 а)  $y'' + 25y = 0$  б)  $9y'' - 6y' + y = 0$  в)  $y'' + 6y' + 8y = 0$
- 3.26 а)  $y'' + 6y' + 10y = 0$  б)  $y'' - 4y' + 4y = 0$  в)  $y'' - 5y' + 4y = 0$
- 3.27 а)  $y'' - 6y = 0$  б)  $4y'' + 8y' - 5y = 0$  в)  $y'' - 6y' + 10y = 0$

**Задача 4.** Найти общее решение дифференциального уравнения.

- 4.1.  $y'' + 2y' + 5y = -\sin 2x$ .      4.2.  $y'' + y = 2\cos 7x + 3\sin 7x$ .  
 4.3.  $y'' - 4y' + 29y = 104\sin 5x$ .      4.4.  $y'' + 5y' = 39\cos 3x - 105\sin 3x$ .  
 4.5.  $y'' + 6y' + 13y = -75\sin 2x$ .      4.6.  $y'' + y = -4\cos x - 2\sin x$ .  
 4.7.  $y'' + 2y' - 24y = 6\cos 3x - 33\sin 3x$ .      4.8.  $y'' - 3y' + 2y = 3\cos x + 19\sin x$ .  
 4.9.  $y'' - 5y' - 6y = 3\cos x + 19\sin x$ .      4.10.  $y'' + y = 2\cos 3x - 3\sin 3x$ .  
 4.11.  $y'' + 2y' + 5y = -2\sin x$ .      4.12.  $y'' + 2y' - 24y = 6\cos 3x - 33\sin 3x$ .  
 4.13.  $y'' + 6y' + 13y = -75\sin 2x$ .      4.14.  $2y'' + 7y' + 3y = 222\sin 3x$ .  
 4.15.  $y'' + y = 2\cos 5x + 3\sin 5x$ .      4.16.  $y'' + 2y' + 5y = -17\sin 2x$ .  
 4.17.  $y'' + 5y' = 39\cos 3x - 105\sin 3x$ .      4.18.  $y'' + 16y = 8\cos 4x$ .  
 4.19.  $y'' - 8y' + 20y = 16(\sin 2x - \cos 2x)$ .  
 4.20.  $y'' - 6y' + 34y = 18\cos 5x + 60\sin 5x$ .  
 4.21.  $y'' + y' - 2y = 9\cos x - 7\sin x$ .      4.22.  $y'' + y = 2\cos 7x - 3\sin 7x$ .  
 4.23.  $y'' + 2y' + 5y = -\cos x$ .      4.24.  $y'' + 2y' + 5y = 10\cos x$ .  
 4.25.  $3y'' - 5y' - 2y = 6\cos 2x + 38\sin 2x$ .  
 4.26.  $y'' - 6y' + 25y = 9\sin 4x - 24\cos 4x$ .  
 4.27.  $4y'' + 3y' - y = 11\cos x - 7\sin x$ .

**Задача 5.** Найти общее решение дифференциального уравнения.

- 5.1.  $y'' + 3y' + 2y = 1 - x^2$       5.2.  $y'' - y' = 3x + 6x^2$   
 5.3.  $y'' - y = x + x^2$       5.4.  $y''' - 3y'' + 3y' - y = 2x$   
 5.5.  $y'' - 6y' + 10y = 51e^{-x}$ .      5.6.  $y'' + 6y' + 10y = 74e^{3x}$ .  
 5.7.  $y'' + 3y' + 2y = 1 + x + x^2$       5.8.  $y'' + 5y' = 72e^{2x}$ .  
 5.9.  $y'' - 8y' + 12y = 36x^4 - 96x^3 + 24x^2 + 16x - 2$ .  
 5.10.  $y'' + 8y' + 25y = 18e^{5x}$       5.11.  $y'' - 9y' + 20y = 126e^{-2x}$   
 5.12.  $y'' + 4y' + 4y = x - x^2$       5.13.  $y'' + 9y = 9x^4 + 12x^2 - 27$   
 5.14.  $y'' + 3y' + 2y = 2x + 3x^2$       5.15.  $y'' - 12y' + 40y = 2e^{6x}$

5.16.  $y'' + 2y' + y = 6e^{-x}$ .

5.17.  $y'' + 2y' + 37y = 37x^2 - 33x + 74$ .

5.18.  $\delta'' + 2\delta' + \delta = -\delta + \delta^2$

5.19.  $6y'' - y' - y = 3e^{2x}$

5.20.  $y'' + 2y' + y = 2 - 3x^2$

5.21.  $y'' - 8y' + 17y = 10e^{2x}$

5.22.  $y'' + 2y' + y = 4x^3 + 24x^2 + 22x - 4$

5.23.  $y'' - 13y' + 12y = x - 1$

5.24.  $y'' + 36y = 36 + 66x - 36x^3$

5.25.  $y'' + 16y = 80e^{2x}$ .

5.26.  $y'' + 4y' = 15e^x$

5.27.  $y'' + 9y = 10e^{3x}$ .

**Задача 6.** Найти общее решение дифференциального уравнения.

6.1.  $y''' - 4y'' + 5y' - 2y = (16 - 12x)e^{-x}$ .

6.2.  $y''' + 5y'' + 7y' + 3y = (16x + 20)e^x$ .

6.3.  $y''' - y'' - y' + y = (3x + 7)e^{2x}$ .

6.4.  $y''' - 2y'' + y' = (2x + 5)e^{2x}$ .

6.5.  $y''' - 3y'' + 4y = (18x - 21)e^{-x}$ .

6.6.  $y''' - 5y'' + 8y' - 4y = (2x - 5)e^x$ .

6.7.  $y''' - 4y'' + 4y' = (x - 1)e^x$ .

6.8.  $y''' + 2y'' + y' = (18x + 21)e^{2x}$ .

6.9.  $y''' + 6y'' + 9y' = (16x + 24)e^x$ .

6.10.  $y''' - 3y' - 2y = -4x \cdot e^x$ .

6.11.  $y''' - 3y' + 2y = (4x + 9)e^{2x}$ .

6.12.  $y''' + 4y'' + 5y' + 2y = (12x + 16)e^x$ .

6.13.  $y''' - y'' - 2y' = (6x - 11)e^{-x}$ .

6.14.  $y''' + y'' - 2y' = (6x + 5)e^x$ .

6.15.  $y''' + 4y'' + 4y' = (9x + 15)e^x$ .

6.16.  $y''' - 3y'' - y' + 3y = (4 - 8x)e^x$ .

6.17.  $y''' - y'' - 4y' + 4y = (7 - 6x)e^x$ .

6.18.  $y''' + 3y'' + 2y' = (1 - 2x)e^{-x}$ .

6.19.  $y''' - 5y'' + 7y' - 3y = (20 - 16x)e^{-x}$ .

6.20.  $y''' - 4y'' + 3y' = -4x \cdot e^x$ .

6.21.  $y''' - 5y'' + 3y' + 9y = (32x - 32)e^{-x}$ .

6.22.  $y''' - 6y'' + 9y' = 4x \cdot e^x$ .

$$6.23. \quad y''' - 7y'' + 15y' - 9y = (8x - 12)e^x.$$

$$6.24. \quad y''' - y'' - 5y' - 3y = -(8x + 4)e^x. \quad 6.25 \quad y''' + 5y'' + 7y' + 3y = (16x + 20)e^x.$$

$$6.26. \quad y''' - 2y'' - 3y' = (8x - 14)e^{-x}. \quad 6.27 \quad y''' + 2y'' - 3y' = (8x + 6)e^x.$$